

Mike Wallace の中間試験 (解答)

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先月号の中間試験の解答を載せる。これは、最高点をとった中村尚君(東北大, 田中研出身)によるものである。

成績は 0.0~4.0 で評価され, 大学院の場合 2.7 以上が合格である。これに単位数の重みを掛けて平均をした GPA (Grade-point average) は, 進学・就職に大きく影響する。

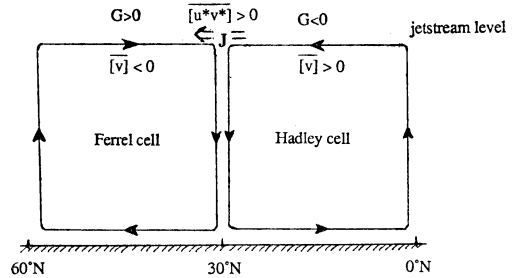
(解答)

- (a) equatorward, (b) equatorward, (c) decrease, (d) a convergence of the eddy flux of zonal momentum, (e) direct, (f) high, (g) Increasing, (h) equatorward, (i) southern, (j) upward, (k) winter, (l) solstices, (m) 30°, (n) 45°, (o) equatorward.

Nations for 2-6.

p: pressure (vertical coordinate); [A]: zonal mean; A*: eddies (deviation from [A]); \bar{A} : time mean; A': transients (deviation from \bar{A}); $A_x = \partial A / \partial x$ (zonal derivative); $A_y = \partial A / \partial y$ (meridional derivative); $A_p = \partial A / \partial p$; u: zonal velocity; v: meridional velocity; (u_g, v_g) : geostrophic flow; ω : vertical velocity in p-coordinate ($=dp/dt$); α : specific volume; Φ : geopotential; f: Coriolis parameter.

- For simplicity the following argument will be made on the "f-plane". A more precise argument should be made in spherical geometry, but the essential part in that case is the same as on the "f-plane". Suppose there are two mean meridional circulation cells, the Hadley and Ferrel cells, as shown in the figure. Poleward eddy transport of zonal momentum $[\overline{u^*v^*}]$ exhibits its



maximum around the jetstream ($\approx 30^\circ\text{N}$, ≈ 200 mb). At the jetstream level convergence of the eddy momentum flux, G, must be balanced by the Coriolis force due to the meridional flow associated with the steady circulation cells (in quasi-geostrophic scaling; friction is negligible). Since the two cells have approximately the same meridional extent (L), $G(|G| \approx [\overline{u^*v^*}]/L)$ is not much different between the two cells at the jetstream level. So approximately

$$\begin{aligned} |(f - [\bar{u}]_y) \cdot [\bar{v}]|_{45^\circ\text{N}} &\approx |(f - [\bar{u}]_y) \cdot [\bar{v}]|_{15^\circ\text{N}} \\ (\text{Ferrel cell}) & \qquad \qquad \qquad (\text{Hadley cell}) \end{aligned}$$

Note that

$$f(45^\circ\text{N}) \approx 1 \times 10^{-4} \text{s}^{-1}; \quad f(15^\circ\text{N}) \approx 0.25 \times 10^{-4} \text{s}^{-1};$$

and that $[\bar{u}]_y$ may be estimated from the velocity of the jetstream:

$$\begin{aligned} [\bar{u}]_y(15^\circ\text{N}) &\approx ([\bar{u}]_{30^\circ\text{N}} - [\bar{u}]_{0^\circ\text{N}}) / (30^\circ \text{ in latitude}) \\ &\approx 30(\text{m/s}) / (3 \times 10^6 \text{m}) = 10^{-5} \text{s}^{-1}; \\ [\bar{u}]_y(45^\circ\text{N}) &\approx ([\bar{u}]_{60^\circ\text{N}} - [\bar{u}]_{30^\circ\text{N}}) / (30^\circ \text{ in latitude}) \\ &\approx -30(\text{m/s}) / (3 \times 10^6 \text{m}) = -10^{-5} \text{s}^{-1}. \end{aligned}$$

Thus

$$\begin{aligned} \frac{|[\bar{v}]|_{15^\circ\text{N}}}{|[\bar{v}]|_{45^\circ\text{N}}} &\approx \frac{|(f - [\bar{u}]_y)_{45^\circ\text{N}}|}{|(f - [\bar{u}]_y)_{15^\circ\text{N}}|} \approx \frac{(10+1) \times 10^{-5}}{(2.5-1) \times 10^{-5}} \\ &= 7.3, \end{aligned}$$

which means that the Hadley cell is several times

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stronger than the Ferrel cell.

3. Since large-scale flow is in quasi-geostrophic balance, $v \approx v_g = \partial \Psi_g / \partial x$ (Ψ_g : geostrophic streamfunction) with errors on the order of 10%. Noting that $[v_g] = [\partial \Psi_g / \partial x] = 0$, we find that $[v]$ is ageostrophic, which is much smaller in magnitude ($\approx 1/10$) than the geostrophic flow. On the other hand, u^* and v^* are dominated by their geostrophic flow components, and then the product u^*v^* does not exhibit significant cancellation along the latitude circles. Therefore $[u^*v^*]$ tends to be estimated much more accurately than $[v]$.

4. In quasi-geostrophic scaling,

$$[v^*\phi^*] = [f^{-1}(\partial\phi^*/\partial x)\phi^*] = f^{-1}[\partial(0.5\phi^{*2})/\partial x] = 0.$$

5. $[\overline{uv}]$: time mean of zonally-averaged total poleward transport of zonal momentum.
 $[\bar{u}][\bar{v}]$: poleward transport of zonal momentum associated with time-averaged, (zonal) mean meridional circulation.
 $[\overline{u'}v']$: time-mean poleward transport of zonal momentum associated with transient mean meridional circulation.
 $[\overline{u^*v^*}]$: time-mean poleward transport of zonal momentum associated with steady and transient eddies.

6. (1) kinematic method:

After a direct measurement of $[v]$, $[\omega]$ is inferred through the zonally averaged continuity equation. However, $[v]$ is quite hard to measure accurately, because $[v]$ is ageostrophic.

- (2) momentum method:

In this method $[v]$ is estimated as a residual of the zonally averaged momentum equation, i.e., $[v] = f^{-1}(\partial[u]/\partial t - G - F)$,

where the convergence of poleward flux of zonal momentum, G , the external force, F , and the momentum tendency, $\partial[u]/\partial t$ have to be measured. Note that no direct measurement of ageostrophic quantities is required. Then the zonally averaged continuity equation is used for the estimation of $[\omega]$. This method is not suitable for in-situ estimation, in which $\partial[u]/\partial t$ is unknown.

- (3) thermodynamic method:

In this method $[\omega]$ is estimated as a residual of the zonally averaged thermodynamic equation, i.e., $[\omega] = \sigma^{-1}(\partial[\alpha]/\partial t - B - Q)$, where the convergence of poleward heat flux, B , the diabatic heating, Q , the stability parameter, σ , and the thermal tendency, $\partial[\alpha]/\partial t$ have to be measured. Note that no ageostrophic quantities are measured directly. Then the zonally averaged continuity equation is applied for the estimation of $[v]$. This method is not suitable for in-situ estimation either.

- (4) ω -equation method:

First, form a diagnostic equation for the streamfunction, χ for $[v]$ and $[\omega]$ by eliminating the tendency terms in the momentum and thermodynamic equations using thermal wind balance. The consequent equation is what is usually called the ω -equation, i.e., $\chi_{yy} + (f^2/\sigma)\chi_{pp} = \sigma^{-1}(B+Q)_y - (f/\sigma)(G+F)_p$, with $[v] = \chi_p$ and $[\omega] = -\chi_y$. No ageostrophic quantities are measured directly. Since no tendency terms appear explicitly, this method can be applied even to in-situ estimation.

月例会「第32回山の気象シンポジウム」のお知らせ

日 時 昭和63年6月18日(土) 13:00から

場 所 気象庁第一会議室(5F)

講演希望の方は演題に200字以内の抽象トを

つけて、4月末日までに、気象庁通報課 岡野光也

(〒100 千代田区大手町 1-3-4)まで郵送して下さい。